Lab#10 - Z Transform in MATLAB

EECS 360: Signal and System Analysis

A very important category of LTI systems is described by difference equations of the following type:

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

From which through Z – Transform we obtain,

$$H(z) = \frac{\sum_{k=0}^{M} b_k Z^{-k}}{\sum_{k=0}^{N} a_k Z^{-k}} = \frac{b_0 + b_1 Z^{-1} + b_2 Z^{-2} + \ldots\ldots + b_M Z^{-M}}{a_0 + a_1 Z^{-1} + a_2 Z^{-2} + \ldots\ldots + a_N Z^{-N}}$$

Here, $H(z)$ is the transfer function of the system.

Assignment 1:

In this part, we will learn how to create pole – zero plot of a given transfer function. The function zplane creates a plot of the positions of zeros and poles in the plane of the complex variable z, with the unit circle for reference, starting from the coefficients a and b. where b and a are row vectors. It uses the function roots to calculate the roots of numerator and denominator of the transfer function.

Create a pole – zero plot for the transfer functions $H(z) = \frac{2 + 2z^{-1} + z^{-2}}{1 - 0.8z^{-1}}$, $H(z) = \frac{1}{1 - 0.9z^{-1}}$, $H(z) = \frac{1 - 0.9^{8} z^{-8}}{1 - 0.9z^{-1}}$

Assignment 2:

In this part, we will learn how to compute the impulse response of a system having coefficients a & b.

[h,t]=impz(b,a) produces the impulse response in vector h and the time axis in vector t. If the output arguments h and t are omitted, a plot of the impulse response is directly displayed. If the impulse response is of infinite length, only its initial part is computed.

Plot the Impulse response for the transfer function $H(z) = \frac{2 + 2z^{-1} + z^{-2}}{1 - 0.8z^{-1}}$, $H(z) = \frac{1}{1 - 0.9z^{-1}}$, $H(z) = \frac{1 - 0.9^{8} z^{-8}}{1 - 0.9z^{-1}}$

Assignment 3:

In this part, we will learn how to compute the frequency response of system expressed by difference equation or rational transfer equation. The function **"freqz"** is used to compute the frequency response.

[H,w]=freqz(b,a,N); where N is a positive integer, returns the frequency response H and the vector w with the N angular frequencies at which H has been calculated (i.e. N equispaced points on the unit circle, between 0 and π). If N is omitted, a default value of 512 is assumed. If no output argument is specified, the amplitude plot and the phase plot of the frequency response are directly displayed.

Plot the magnitude and phase response for the transfer functions $H(z)=\dfrac{2+2z^{-1}+z^{-2}}{1-0.8z^{-1}}$,

$H(z)=\dfrac{1}{1-0.9z^{-1}}$, $H(z)=\dfrac{1-0.9^{8}z^{-8}}{1-0.9z^{-1}}$

Assignment 4:

In this part we will learn about partial fraction expansion. If a given transfer function looks like as follows:

$$H(z)=\frac{b_0+b_1Z^{-1}+b_2Z^{-2}+......+b_MZ^{-M}}{a_0+a_1Z^{-1}+a_2Z^{-2}+......+a_NZ^{-N}}$$

$H(z)$ can be expressed by means of partial fraction expansion as

$$H(z)=\sum_{k=0}^{M-N}C_kZ^{-k}+\sum_{k=1}^{N}\frac{A_k}{(1-P_kZ^{-1})}$$

We can derive such a partial fraction expansion by means of the Matlab function residuez.
[A,p,C]=residuez(b,a) computes the constants on the numerator (A_k, known as also residues), poles (P_k), and direct terms (C_k) of $H(z)$. The returned column vector A contains the residues, column vector p contains the pole locations, and row vector C contains the polynomials terms when M≥N.

Compute and write the partial fraction expansion for the transfer function

$H(z)=\dfrac{z^{-1}}{3-4z^{-1}+z^{-2}}$, $H(z)=\dfrac{1-0.64z^{-2}}{1-0.2z^{-1}-0.08z^{-2}}$

Partial Expansion form of $1^{st}H(z)$ =

Partial Expansion form of 2nd $H(z)$ =

Assignment 5:

Design a low pass filter using poles ($0.5,\ 0.45+0.5i,\ 0.45-0.5i$) and zeros ($-1,\ i, -i$) and Convert

I. the pole-zero representation to a rational transfer function representation
II. Make a plot of the desired magnitude and phase response

Tips for Lab Assignment 5:

Sample Code:

```
% Assign your poles as row vector
% Assign your zeros as row vector
% use K=1

% [b,a]=zp2tf(z,p,k)

% [H,W]=freqz(b,a)

% plot magnitude of H

% plot phase of H
```

N.B: The command [b,a]=zp2tf(z,p,k) finds the coefficients b and a of the associated transfer function, given a set of zero locations in vector z, a set of pole locations in vector p, and a gain in scalar k. We define the gain k=1 for this code.

You can check your result by using command zplane(b,a). This command will plot a pole – zero curve that exactly positions poles and zeros that you defined at the beginning of this code.

Questions:

1. How Z transform and Laplace transform are related by?
2. H (z) is discrete rational transfer function. To ensure that both H(z) and its inverse are stable, where the poles and zeros should be placed in the unit circle?
3. What is the set of all values of z for which X(z) attains a finite value?
4. What is the ROC of the system function H(z) if the discrete time LTI system is BIBO stable?
5. What is the ROC of z-transform of finite duration anti-causal sequence?